# STAT 131 — Discussion (Week 2)

## Notation and formulas

**Events as sets.**

- $A \cup B =$ "$A$ or $B$"
- $A \cap B =$ "$A$ and $B$"
- $A^c =$ "not $A$"

**Equally likely outcomes.**

If $\mathcal{S}$ is finite and all outcomes are equally likely, $Pr(A) = \frac{\text{total \# outcomes in } A}{\text{total \# of outcomes in } \mathcal{S}}$.

**Identities.**

- $Pr(A^c) = 1 - Pr(A)$
- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$
- If $A \cap B = \varnothing$ (disjoint), then $Pr(A \cup B) = Pr(A) + Pr(B)$.

**De Morgan (rewrite complements).**

$$(A \cup B)^c = A^c \cap B^c, \qquad (A \cap B)^c = A^c \cup B^c.$$

**Uniform on a region (geometric probability).**

If a point is uniform on $\mathcal{S} \subset \mathbb{R}^2$, then $Pr(E) = \frac{\text{Area}(E)}{\text{Area}(\mathcal{S})}$.

**What to write first (workflow).**

- Define $\mathcal{S}$ and the events (as sets).
- Choose the tool: counting vs. identities vs. area ratio.

**Version 1**

## Problem 1 Sample space, events, and identities

Toss a fair coin twice, then roll a fair six-sided die. Let $\mathcal{S}$ be the sample space. Define $A =$ "at least one head" and $B =$ "the die shows an even number."

(a) Give a valid $\mathcal{S}$ and compute the size of $\mathcal{S}$.

(b) Write $A$ and $B$ explicitly as subsets of $\mathcal{S}$.

(c) Compute $Pr(A)$, $Pr(B)$, $Pr(A \cap B)$, and $Pr(A \cup B)$.

(d) Compute $Pr((A \cup B)^c)$ using De Morgan's law (and check with a complement).

## Problem 2 Set notation + geometric probability

(a) A system fails if at least one of three components fails. Let $F_i$ be the event "component $i$ fails." Write the failure event $F$ and $F^c$ using $\cup, \cap, (\cdot)^c$.

(b) A point $(X, Y)$ is chosen uniformly on $\mathcal{S} = \{(x, y) : 0 \leq x \leq 4,\ 0 \leq y \leq 3\}$. Let $E = \{(x, y) \in \mathcal{S} : x \geq 1,\ y \geq 2\}$. Compute $Pr(E)$.

(c) Write $E^c$ using set notation and compute $Pr(E^c)$ (then verify $Pr(E) + Pr(E^c) = 1$).

**Version 2**

# Problem 1 Sample space, events, and identities

Toss a fair coin twice, then roll a fair six-sided die. Let $\mathcal{S}$ be the sample space. Define $A =$ "at least one head" and $B =$ "the die shows a multiple of 3."

(a) Give a valid $\mathcal{S}$ and compute the size of $\mathcal{S}$.

(b) Write $A$ and $B$ explicitly as subsets of $\mathcal{S}$.

(c) Compute $Pr(A)$, $Pr(B)$, $Pr(A \cap B)$, and $Pr(A \cup B)$.

(d) Compute $Pr((A \cup B)^c)$ using De Morgan's law (and check with a complement).

# Problem 2 Set notation + geometric probability

(a) A system fails if at least one of three components fails. Let $F_i$ be the event "component $i$ fails." Write the failure event $F$ and $F^c$ using $\cup, \cap, (\cdot)^c$.

(b) A point $(X, Y)$ is chosen uniformly on $\mathcal{S} = \{(x, y) : 0 \leq x \leq 5,\ 0 \leq y \leq 4\}$. Let $E = \{(x, y) \in \mathcal{S} : x \geq 2,\ y \geq 3\}$. Compute $Pr(E)$.

(c) Write $E^c$ using set notation and compute $Pr(E^c)$ (then verify $Pr(E) + Pr(E^c) = 1$).