

STAT 131 — Discussion (Week 3)

Notation and formulas

Simple finite sample spaces. If all outcomes in $\mathcal{S}$ are equally likely, then

$$Pr(E) = \frac{|E|}{|\mathcal{S}|}.$$

Multiplication principle. If an experiment has k stages with $n_1, n_2, \dots, n_k$ choices, then the total number of outcomes is

$$n_1 n_2 \cdots n_k.$$

Permutations and combinations.

$$P_{n,k} = \frac{n!}{(n-k)!} \quad (\text{order matters}),$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} \quad (\text{order does not matter}).$$

Multinomial coefficient. If $n_1 + n_2 + \cdots + n_r = n$, then

$$\binom{n}{n_1, n_2, \dots, n_r} = \frac{n!}{n_1! n_2! \cdots n_r!}.$$

Disjoint cases and conditional probability. If $A_1, \dots, A_m$ are disjoint, then

$$Pr\left(\bigcup_{i=1}^m A_i\right) = \sum_{i=1}^m Pr(A_i).$$

If $Pr(B) > 0$, then

$$Pr(A | B) = \frac{Pr(A \cap B)}{Pr(B)}.$$

What to ask first.

- Are all outcomes equally likely?
- Does *order* matter?
- Are we conditioning on extra information?

Version 1

Write the counting formula you use before simplifying. A short setup is enough.

Problem 1 Counting: when does order matter?

A student organization has 10 members.

- (a) In how many ways can it elect a president, a vice-president, and a treasurer?
- (b) In how many ways can it choose a 3-person welcome committee with no roles?
- (c) In how many ways can it assign all 10 members to three labeled teams:

Outreach (4), Publicity (3), Logistics (3)?

Problem 2 Probability by counting

A fair coin is tossed 6 times. Define

$A = \{\text{exactly 2 heads}\}, \quad B = \{\text{at most 2 heads}\}, \quad C = \{\text{an even number of heads}\}.$

- (a) Compute the total number of outcomes in the sample space.
- (b) Compute $Pr(A)$.
- (c) Compute $Pr(B)$. Write it as a sum of disjoint cases.
- (d) Compute $Pr(B \mid C)$.

Version 2

Write the counting formula you use before simplifying. A short setup is enough.

Problem 1 Counting: when does order matter?

A student organization has 11 members.

- (a) In how many ways can it elect a president, a vice-president, and a treasurer?
- (b) In how many ways can it choose a 4-person welcome committee with no roles?
- (c) In how many ways can it assign all 11 members to three labeled teams:

Outreach (5), Publicity (3), Logistics (3)?

Problem 2 Probability by counting

A fair coin is tossed 7 times. Define

$A = \{\text{exactly 3 heads}\}, \quad B = \{\text{at most 3 heads}\}, \quad C = \{\text{an odd number of heads}\}.$

- (a) Compute the total number of outcomes in the sample space.
- (b) Compute $Pr(A)$.
- (c) Compute $Pr(B)$. Write it as a sum of disjoint cases.
- (d) Compute $Pr(B \mid C)$.