STAT 131 — Discussion (Week 5)

Notation and formulas

Discrete models (choose the right one).

- **Binomial:** $X \sim \mathrm{Bin}(n, p) = \#$ successes in n independent trials.

$$Pr(X = k) = \binom{n}{k} p^k (1-p)^{n-k}, \qquad k = 0, 1, \ldots, n.$$

- **Geometric:** $Y = \#$ failures before first success (independent trials, success prob. p).

$$Pr(Y = k) = (1-p)^k p, \qquad k = 0, 1, 2, \ldots$$

  and $Pr(Y \geq k) = (1-p)^k$.
- **Negative Binomial:** $Z = \#$ failures before r successes.

$$Pr(Z = k) = \binom{k+r-1}{k} (1-p)^k p^r, \qquad k = 0, 1, 2, \ldots$$

- **Hypergeometric:** sampling *without replacement*. If A success items and B failure items, sample n, then

$$Pr(X = k) = \frac{\binom{A}{k}\binom{B}{n-k}}{\binom{A+B}{n}}.$$

- **Poisson:** $X \sim \mathrm{Pois}(\lambda)$ counts in a fixed interval (rate λ).

$$Pr(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}, \qquad k = 0, 1, 2, \ldots$$

  Often use complements: $Pr(X \geq 2) = 1 - Pr(X = 0) - Pr(X = 1)$.

Cdf, continuous probabilities, and quantiles.

- **Cdf:** $F(x) = Pr(X \leq x)$. Discrete: $F(x) = \sum_{t \leq x} Pr(X = t)$. Continuous: $F(x) = \int_{-\infty}^{x} f(t)\,dt$ with $f(x)$ the pdf (or density) of X.
- **Continuous:** $P(X = x) = 0$ and (when $a \leq b$) $P(a \leq X \leq b) = F(b) - F(a)$.
- **Quantile:** continuous $q_p = F^{-1}(p)$ (solve $F(q_p) = p$). Discrete: smallest x with $F(x) \geq p$.

What to write first (workflow).

- Identify the experiment: independent trials vs. without replacement vs. counting in time.
- Write the correct pmf/pdf, then compute the requested probability.
- If cdf/quantile: write $F(x)$ piecewise, then use $F(b) - F(a)$ or solve $F(q) = p$.

Version 1

Problem 1 Choose the right discrete model

Each part is a different experiment. State the model you are using (Binomial / Geometric / Neg-Bin / Hypergeometric / Poisson), then compute.

(a) Independent trials with success probability $p = 0.6$. Let X be the number of successes in $n = 8$ trials. Compute $Pr(X = 5)$.

(b) Independent trials with success probability $p = 0.6$. Let Y be the number of failures before the first success. Compute $Pr(Y = 3)$ and $Pr(Y \geq 3)$.

(c) **Sampling without replacement: a box has 12 defective and 88 non-defective batteries. You sample $n = 5$ without replacement. Let H = number of defectives in the sample. Compute $Pr(H = 2)$.

(d) **Poisson counts: hits occur with rate $\lambda = 3$ per minute. Let N be the number of hits in 1 minute. Compute $Pr(N \geq 2)$ using a complement.

Problem 2 Continuous cdf + probabilities + quantiles

(a) (****Uniform**) Let $X \sim \text{Unif}[-2, 3]$. Compute $Pr(-1 \leq X \leq 2)$ and write $F(x)$ piecewise.

(b) (**Normalizing + cdf**) A continuous r.v. X has density

$$f_X(x) = \begin{cases} c\,x, & 0 < x \leq 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find c, then compute $Pr(1 \leq X \leq 2)$ and write $F_X(x)$ piecewise.

(c) (**Quantiles**) For $X \sim \text{Unif}[-2, 3]$, find the median and the $p = 0.25$ quantile.

Version 2

Problem 1 Choose the right discrete model

Each part is a different experiment. State the model you are using (Binomial / Geometric / Neg-Bin / Hypergeometric / Poisson), then compute.

(a) Independent trials with success probability $p = 0.55$. Let X be the number of successes in $n = 10$ trials. Compute $Pr(X = 6)$.

(b) Independent trials with success probability $p = 0.55$. Let Y be the number of failures before the first success. Compute $Pr(Y = 2)$ and $Pr(Y \geq 2)$.

(c) Sampling without replacement: a box has 10 defective and 90 non-defective batteries. You sample $n = 5$ without replacement. Let H = number of defectives in the sample. Compute $Pr(H = 1)$.

(d) Poisson counts: hits occur with rate $\lambda = 2$ per minute. Let N be the number of hits in 1 minute. Compute $Pr(N \geq 2)$ using a complement.

Problem 2 Continuous cdf + probabilities + quantiles

(a) (**Uniform**) Let $X \sim \text{Unif}[-1, 5]$. Compute $Pr(0 \leq X \leq 3)$ and write $F(x)$ piecewise.

(b) (**Normalizing + cdf**) A continuous r.v. X has density

$$f_X(x) = \begin{cases} c\,x, & 0 < x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find c, then compute $Pr(1 \leq X \leq 2)$ and write $F_X(x)$ piecewise.

(c) (**Quantiles**) For $X \sim \text{Unif}[-1, 5]$, find the median and the $p = 0.75$ quantile.