STAT 131 — Discussion (Week 6)

Notation and formulas (joint distributions: events from the joint)

1) Joint distribution.

- **Discrete joint pmf:**

$$p_{X,Y}(x,y) = \Pr(X = x,\, Y = y), \qquad \sum_x \sum_y p_{X,Y}(x,y) = 1.$$

- **Continuous joint pdf:**

$$f_{X,Y}(x,y) \geq 0, \qquad \iint_{\mathbb{R}^2} f_{X,Y}(x,y)\,dx\,dy = 1.$$

2) Probabilities from the joint.

- **Discrete: sum the cells in the event.** For an event E made of possible pairs,

$$\Pr\big((X,Y) \in E\big) = \sum_{(x,y)\in E} p_{X,Y}(x,y).$$

- **Continuous: integrate over the region.** For a region R in the support,

$$\Pr\big((X,Y) \in R\big) = \iint_R f_{X,Y}(x,y)\,dx\,dy.$$

3) Translating bivariate events. Common events are inequalities involving both variables:

$$\{Y < X\}, \qquad \{X + Y > 1\}, \qquad \{X > 3/4,\, Y < 1/2\}.$$

The main step is to translate the event into **cells** or **bounds**.

4) Continuous regions: two common slicing strategies. For a rectangle support, choose whichever order gives simpler bounds.

$$\Pr(Y < X) = \iint_{\{(x,y):y<x\}} f_{X,Y}(x,y)\,dx\,dy.$$

For example, on $0 < x < 1$, $0 < y < 2$,

$$\{Y < X\}: \qquad 0 < x < 1, \quad 0 < y < x.$$

For the same support,

$$\{X + Y > 1\}: \qquad 0 < x < 1, \quad 1 - x < y < 2.$$

5) Important continuous-variable reminder.

$$\Pr(X = x, Y = y) = 0.$$

For continuous variables, probability comes from an **area/region integral**, not from the value of the density at a point.

Version 1

Problem 1 Discrete joint pmf: translate events into cells

Let $X \in \{0, 1\}$ and $Y \in \{0, 1, 2\}$. The joint pmf is given by:

	$X = 0$	$X = 1$
$Y = 0$	0.12	0.08
$Y = 1$	0.18	0.12
$Y = 2$	0.30	0.20

For each part, first write the event as a set of cells, then add the corresponding probabilities.

(a) Compute $\Pr(Y > X)$.

(b) Compute $\Pr(X + Y \geq 2)$.

(c) Compute $\Pr(X = 1 \text{ or } Y = 0)$.

(d) Compute $\Pr(Y \geq 1,\ X + Y \leq 2)$.

Problem 2 Continuous joint density: translate events into regions

Let the joint density be

$$f_{X,Y}(x, y) = \begin{cases} x, & 0 < x < 1,\ 0 < y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

For each part, sketch the support and shade the event region before writing the integral.

(a) Compute $\Pr(Y < X)$.

(b) Compute $\Pr(X + Y > 1)$.

(c) Compute $\Pr(X > 3/4,\ Y < 1/2)$.

Version 2

Problem 1 Discrete joint pmf: translate events into cells

Let $X \in \{0, 1\}$ and $Y \in \{0, 1, 2\}$. The joint pmf is given by:

	$X = 0$	$X = 1$
$Y = 0$	0.10	0.15
$Y = 1$	0.25	0.05
$Y = 2$	0.35	0.10

For each part, first write the event as a set of cells, then add the corresponding probabilities.

(a) Compute $\Pr(Y > X)$.

(b) Compute $\Pr(X + Y \geq 2)$.

(c) Compute $\Pr(X = 0 \text{ or } Y = 0)$.

(d) Compute $\Pr(Y \geq 1,\ X + Y \leq 2)$.

Problem 2 Continuous joint density: translate events into regions

Let the joint density be

$$f_{X,Y}(x, y) = \begin{cases} y, & 0 < x < 2,\ 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

For each part, sketch the support and shade the event region before writing the integral.

(a) Compute $\Pr(Y < X)$.

(b) Compute $\Pr(X + Y > 2)$.

(c) Compute $\Pr(X < 1,\ Y > 1/2)$.