STAT 131 — Discussion (Week 7)

Notation and formulas (joint distributions + conditioning)

1) Joint distribution (pmf vs. pdf).

- **Discrete joint pmf:**

$$p_{X,Y}(x,y) = Pr(X = x,\ Y = y), \quad \sum_x \sum_y p_{X,Y}(x,y) = 1$$

- **Continuous joint pdf:**

$$f_{X,Y}(x,y) \geq 0, \quad \int\int_{\mathbb{R}^2} f_{X,Y}(x,y)\,dx\,dy = 1$$

2) Getting probabilities from the joint.

- **Discrete (sum points):** for an event E (set of points),

$$Pr\big((X,Y) \in E\big) = \sum_{(x,y)\in E} p_{X,Y}(x,y).$$

- **Continuous (integrate region):** for a region R,

$$Pr\big((X,Y) \in R\big) = \iint_R f_{X,Y}(x,y)\,dx\,dy, \qquad \text{and } Pr(X = x,\ Y = y) = 0.$$

3) Marginals ("sum/integrate out" the other variable).

$$\text{Discrete:} \quad p_X(x) = \sum_y p_{X,Y}(x,y), \qquad p_Y(y) = \sum_x p_{X,Y}(x,y).$$

$$\text{Continuous:} \quad f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy, \qquad f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx.$$

4) Conditional distributions ("normalize by the marginal").

- **Discrete:** $p_{X|Y}(x \mid y) = Pr(X = x \mid Y = y) = \dfrac{p_{X,Y}(x,y)}{p_Y(y)}$ (when $p_Y(y) > 0$).
- **Continuous:** $f_{X|Y}(x \mid y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)}$ (when $f_Y(y) > 0$).

5) Independence (same test in both cases).

$$X \perp Y \quad \Longleftrightarrow \quad \text{joint} = (\text{marginal of } X) \times (\text{marginal of } Y) \text{ on the support.}$$

$$\text{Discrete: } p_{X,Y}(x,y) = p_X(x)p_Y(y), \qquad \text{Continuous: } f_{X,Y}(x,y) = f_X(x)f_Y(y).$$

If independent, then for sets A, B,

$$Pr(X \in A,\ Y \in B) = Pr(X \in A)\,Pr(Y \in B).$$

Version 1

Problem 1

Let $X \in \{0, 1\}$ and $Y \in \{0, 1, 2\}$. The joint pmf is given by:

	$X = 0$	$X = 1$
$Y = 0$	0.12	0.08
$Y = 1$	0.18	0.12
$Y = 2$	0.30	0.20

Hint: compute marginals first; then conditionals are "divide by the right marginal."

(a) Compute the marginals $f_X(0), f_X(1)$.

(b) Compute $Pr(Y \geq 1 \mid X = 0)$.

Problem 2

Let the joint density be

$$f_{X,Y}(x, y) = \begin{cases} x, & 0 < x < 1,\ 0 < y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the marginals $f_X(x)$ (include its supports).

(b) Find the conditional density $f_{X|Y}(x \mid y)$ for $0 < y < 2$ (and state its support in x). Does it depend on y? Based on your previous answer, are they independent?

Version 2

Problem 1

Let $X \in \{0, 1\}$ and $Y \in \{0, 1, 2\}$. The joint pmf is given by:

	$X = 0$	$X = 1$
$Y = 0$	0.10	0.15
$Y = 1$	0.25	0.05
$Y = 2$	0.35	0.10

Hint: marginals first; then conditionals are "divide by the right marginal."

(a) Compute the marginals $f_X(0), f_X(1)$.

(b) $Pr(Y = 2 \mid X = 1)$.

Problem 2

Let the joint density be

$$f_{X,Y}(x, y) = \begin{cases} y, & 0 < x < 2,\ 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the marginals $f_X(x)$ (include its supports).

(b) Find the conditional density $f_{Y|X}(y \mid x)$ for $0 < x < 2$ (and state its support in y). Does it depend on x? Based on your previous answer, are they independent?