

STAT 131 — Discussion (Week 9)

Notation and formulas (Week 9)

Variance and SD.

$$\text{Var}(X) = E[(X - EX)^2] = E[X^2] - (E[X])^2, \quad \text{SD}(X) = \sqrt{\text{Var}(X)}.$$

$$E[aX + b] = aE[X] + b, \quad \text{Var}(aX + b) = a^2 \text{Var}(X).$$

Covariance and correlation.

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y], \quad \rho(X, Y) = \frac{\text{Cov}(X, Y)}{\text{SD}(X) \text{SD}(Y)} \quad (\text{SD} > 0).$$

$$\text{Var}(aX + bY + c) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) + 2ab \text{Cov}(X, Y).$$

Conditional expectation/variance and “total” laws.

$$E(Y | X = x) = \sum_y y p_{Y|X}(y | x) \quad \text{or} \quad E(Y | X = x) = \int y f_{Y|X}(y | x) dy,$$

$$\text{Var}(Y | X = x) = E(Y^2 | X = x) - [E(Y | X = x)]^2.$$

$$E(Y) = E(E(Y | X)), \quad \text{Var}(Y) = E(\text{Var}(Y | X)) + \text{Var}(E(Y | X)).$$

Standard normal. $Z \sim N(0, 1)$ has CDF $\Phi(z) = P(Z \leq z)$ and symmetry

$$\Phi(-z) = 1 - \Phi(z) \quad (z > 0).$$

Version 1

Note: TA will help with Problem 1a), 1c), and Problem 2b).

Problem 1

Let $X \in \{0, 1\}$ and $Y \in \{0, 2\}$ with joint pmf:

	$X = 0$	$X = 1$
$Y = 0$	0.375	0.125
$Y = 2$	0.125	0.375

- (a) Compute $E[X]$, $E[Y]$, and $E[XY]$.
- (b) Compute $\text{Var}(X)$ and $\text{Var}(Y)$.
- (c) Compute $\text{Cov}(X, Y)$ and $\rho(X, Y)$.
- (d) Compute $\text{Var}(2X - Y + 3)$.

Hint: since $X \in \{0, 1\}$, $X^2 = X$.

Problem 2

Let $P \sim \text{Uniform}(0, 1)$. Given $P = p$, let

$$Y \mid P = p \sim \text{Bin}(n = 12, p).$$

- (a) Compute $E(Y \mid P = p)$ and $\text{Var}(Y \mid P = p)$.
- (b) Compute $E(Y)$ using $E(Y) = E(E(Y \mid P))$.
- (c) Compute $\text{Var}(Y)$ using $\text{Var}(Y) = E(\text{Var}(Y \mid P)) + \text{Var}(E(Y \mid P))$.

Version 2

Problem 1

Let $X \in \{0, 1\}$ and $Y \in \{0, 2\}$ with joint pmf:

	$X = 0$	$X = 1$
$Y = 0$	0.125	0.375
$Y = 2$	0.375	0.125

- (a) Compute $E[X]$, $E[Y]$, and $E[XY]$.
 - (b) Compute $\text{Var}(X)$ and $\text{Var}(Y)$.
 - (c) Compute $\text{Cov}(X, Y)$ and $\rho(X, Y)$.
 - (d) Compute $\text{Var}(2X - Y + 3)$.
- Hint:* $X^2 = X$ and $Y^2 = 4$ when $Y = 2$.

Problem 2

Let $P \sim \text{Uniform}(0, 1)$. Given $P = p$, let

$$Y \mid P = p \sim \text{Bin}(n = 10, p).$$

- (a) Compute $E(Y \mid P = p)$ and $\text{Var}(Y \mid P = p)$.
- (b) Compute $E(Y)$ using $E(Y) = E(E(Y \mid P))$.
- (c) Compute $\text{Var}(Y)$ using $\text{Var}(Y) = E(\text{Var}(Y \mid P)) + \text{Var}(E(Y \mid P))$.