STAT 131 – Final Review

Workflows, formulas, and representative practice

Antonio Aguirre

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A cumulative review of the main STAT 131 problem-solving tools.

How to use this final review

Purpose

This review is a study guide for the course workflows: identify the random variable, write the support, choose the tool, and compute with clear justification.

Before computing, ask

1. What random variable is being used or created?
2. Is it discrete, continuous, a sum, a maximum/minimum, or a conditional model?
3. What is the support?
4. Do we need a probability, a CDF/PDF/PMF, an expectation, or an approximation?

Fair review rule

These examples review core course skills. They are not predictions of final-exam questions.

What a strong solution looks like

Write the setup before the arithmetic

A complete solution usually has four visible pieces:

1. **Random variable/event:** define what is being computed.
2. **Support or bounds:** list possible values or draw the region.
3. **Tool:** name the formula, theorem, or approximation.
4. **Conclusion:** simplify and state the result.

Partial-credit habit

Show the setup. Unsupported numerical answers are hard to grade, even when the number is close.

Good habits

- CDF/density: state the support first.
- CLT/normal: show the standardization.
- Convolution/integrals: show the limits.
- Expectation: check whether LOTUS or total expectation is faster.

Quick check

Before moving on, ask: did I define the right variable and event?

Final review map

One habit across the course

words $\longrightarrow$ random variable $\longrightarrow$ support $\longrightarrow$ tool.

Core tools

Topic	**Action**
Probability rules	condition, total probability, Bayes
Transformations	CDF recipe or one-to-one theorem
Joint densities	draw support, integrate over region
Order statistics	translate max/min events
Expectations	use LOTUS or total expectation
CLT/convolution	standardize averages or integrate over valid line segments

read the question
↓
define the random variable
↓
state the support
↓
choose exact / bound / approximation
↓
compute and justify

Probability identities that keep appearing

Basic rules

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B).$$

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}, \quad \Pr(B) > 0.$$

$$\Pr(A \cap B) = \Pr(A \mid B)\Pr(B).$$

Independence check

$$A \perp\!\!\!\perp B \iff \Pr(A \cap B) = \Pr(A)\Pr(B).$$

Total probability and Bayes

If $B_1, \ldots, B_k$ form a partition, then

$$\Pr(A) = \sum_{j=1}^{k} \Pr(A \mid B_j)\Pr(B_j).$$

$$\Pr(B_i \mid A) = \frac{\Pr(A \mid B_i)\Pr(B_i)}{\sum_{j=1}^{k} \Pr(A \mid B_j)\Pr(B_j)}.$$

Reading cue

Conditioning changes the reference set. Always identify the event after the vertical bar.

Short-calculation toolbox

Common one-line tools

Task	Formula / setup
Correlation	$\mathrm{Corr}(X, Y) = \dfrac{\mathrm{Cov}(X, Y)}{\mathrm{SD}(X)\mathrm{SD}(Y)}$
Variance of linear combination	$\mathrm{Var}(aX + bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y) + 2ab\mathrm{Cov}(X, Y)$
Independent sum variance	$\mathrm{Var}\left(\sum_i a_i X_i\right) = \sum_i a_i^2 \mathrm{Var}(X_i)$
Poisson rate scaling	if rate is λ per unit, count over t units is $\mathrm{Poisson}(\lambda t)$
Normal linear combination	independent normal variables stay normal under linear combinations

Exam habit

For each part, write the formula before substituting numbers. This makes partial credit possible and catches wrong variables.

Poisson scaling: counts over time or space

Rule

If events occur according to a Poisson model with rate λ per unit, then the count over t units has distribution

$$N(t) \sim \text{Poisson}(\lambda t).$$

Thus

$$\Pr(N(t) = k) = e^{-\lambda t}\frac{(\lambda t)^k}{k!}.$$

Common mistake

When the time or area changes, the mean changes too.

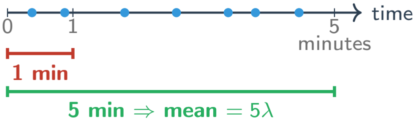
Representative example

Suppose a sensor records events at rate 2 per minute. Let Y be the number of events in 5 minutes. Then

$$Y \sim \text{Poisson}(10).$$

So

$$\Pr(Y = 3) = e^{-10}\frac{10^3}{3!}.$$



Normal linear combinations

Core result

If $X_1, \ldots, X_n$ are independent and

$$X_i \sim N(\mu_i, \sigma_i^2),$$

then

$$\sum_{i=1}^{n} a_i X_i \sim N\left(\sum_{i=1}^{n} a_i \mu_i,\ \sum_{i=1}^{n} a_i^2 \sigma_i^2\right).$$

Mean

Coefficients keep their sign:

$$\mathbb{E}(aX - bY) = a\mathbb{E}(X) - b\mathbb{E}(Y).$$

Variance

Coefficients get squared:

$$\mathrm{Var}(aX - bY) = a^2\mathrm{Var}(X) + b^2\mathrm{Var}(Y)$$

when $X \perp\!\!\!\perp Y$.

Use the standard deviation, not the variance, when standardizing a normal random variable.

Normal probabilities: derive, then standardize

Workflow

First derive the distribution of the target variable. If

$$W \sim N(\mu_W, \sigma_W^2),$$

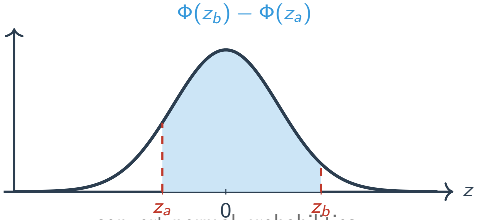
then standardize with

$$Z = \frac{W - \mu_W}{\sigma_W} \sim N(0, 1).$$

For endpoints $a < b$,

$$\Pr(a < W < b) = \Phi\left(\frac{b - \mu_W}{\sigma_W}\right) - \Phi\left(\frac{a - \mu_W}{\sigma_W}\right).$$

Reminder: standardize by the standard deviation σ_W, not the variance.



convert normal probabilities
to left-tail table values

Right tail

$$\Pr(W > w) = 1 - \Phi\left(\frac{w - \mu_W}{\sigma_W}\right).$$

Transformations: the CDF recipe

Use this when $Y = g(X)$ and X is continuous

1. Start with X: find $\mathrm{Supp}(X)$ and compute

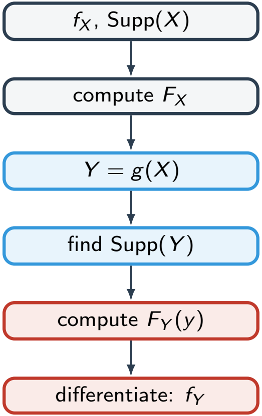
$$F_X(x) = \Pr(X \le x) = \int_{-\infty}^{x} f_X(t)\,\mathrm{d}t.$$

2. Transform the support: find $\mathrm{Supp}(Y)$ from the range of g.
3. Write the transformed CDF:

$$F_Y(y) = \Pr(Y \le y) = \Pr(g(X) \le y).$$

4. Rewrite the event in terms of X and use F_X.
5. Differentiate on the support:

$$f_Y(y) = F_Y'(y).$$



The CDF method is the safest method, including many-to-one transformations.

Transformation example: CDF method

Let

$$X \sim \mathrm{Unif}(0,1), \;\; Y = -\log(1-X).$$

Support and CDF

Since $0 < X < 1$,

$$Y = -\log(1-X) > 0, \;\; \mathrm{Supp}(Y) = (0,\infty).$$

For $y > 0$,

$$F_Y(y) = \Pr(-\log(1-X) \le y).$$

$$= \Pr(X \le 1 - e^{-y}) = F_X(1 - e^{-y}).$$

Since $F_X(x) = x$ on $(0,1)$,

$$F_Y(y) = 1 - e^{-y}.$$

Density

$$f_Y(y) = F_Y'(y) = e^{-y}, \;\; y > 0.$$

Thus

$$Y \sim \mathrm{Exp}(1).$$

What to notice

The support changes from $(0,1)$ to $(0,\infty)$. Do not skip that step.

Shortcut theorem: only after the support check

One-to-one transformation theorem

If $Y = g(X)$, g is one-to-one on $\text{Supp}(X)$, and

$$s(y) = g^{-1}(y),$$

then

$$f_Y(y) = f_X(s(y))\,|s'(y)|, \;\; y \in \text{Supp}(Y).$$

Checklist

1. Verify one-to-one on $\text{Supp}(X)$.
2. Find the inverse $s(y)$.
3. Find $\text{Supp}(Y)$.
4. Multiply by $|s'(y)|$.

Small example

Let $X \sim \mathrm{Unif}(0,1)$ and $Y = X^2$. On $(0,1)$, the map is one-to-one.

$$s(y) = \sqrt{y}, \;\; |s'(y)| = \frac{1}{2\sqrt{y}}.$$

Since $\text{Supp}(Y) = (0,1)$,

$$f_Y(y) = \frac{1}{2\sqrt{y}}, \;\; 0 < y < 1.$$

Many-to-one warning

Example

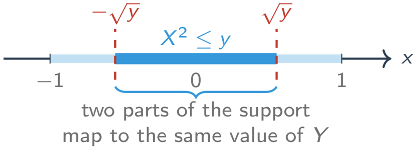
Let

$$X \sim \mathrm{Unif}(-1,1), \;\; Y = X^2.$$

Now $x \mapsto x^2$ is not one-to-one on $(-1,1)$. For $0 < y < 1$,

$$F_Y(y) = \Pr(X^2 \le y) = \Pr(-\sqrt{y} \le X \le \sqrt{y}).$$

$$F_Y(y) = F_X(\sqrt{y}) - F_X(-\sqrt{y}).$$



Rule

If a transformation is many-to-one, use the CDF method or split into one-to-one branches.

Maximum and minimum: general formulas

Let $X_1, \ldots, X_n$ be i.i.d. with CDF F and density f.

Maximum

Let
$$M_n = \max\{X_1, \ldots, X_n\}.$$
Then
$$\{M_n \le y\} = \{X_1 \le y, \ldots, X_n \le y\}.$$
Thus
$$F_{M_n}(y) = [F(y)]^n,$$
$$f_{M_n}(y) = n[F(y)]^{n-1} f(y).$$

Minimum

Let
$$L_n = \min\{X_1, \ldots, X_n\}.$$
Use the complement:
$$\{L_n > y\} = \{X_1 > y, \ldots, X_n > y\}.$$
Thus
$$F_{L_n}(y) = 1 - [1 - F(y)]^n,$$
$$f_{L_n}(y) = n[1 - F(y)]^{n-1} f(y).$$

Max/min visual cue

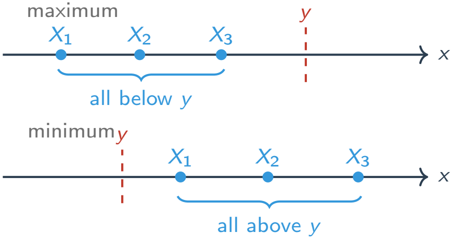
For a maximum

$$M_n \leq y \iff X_i \leq y \text{ for all } i.$$

For a minimum

$$L_n > y \iff X_i > y \text{ for all } i.$$

Then use the complement to get $F_{L_n}(y)$.



Memory cue

Maximum: **all below**. Minimum: **all above**, then subtract from 1.

Order-statistic example

Let

$$X_1, X_2, X_3 \overset{\text{iid}}{\sim} \mathrm{Unif}(0,1).$$

Maximum $M = \max X_i$

For $0 < y < 1$,

$$F_M(y) = y^3, \; f_M(y) = 3y^2.$$

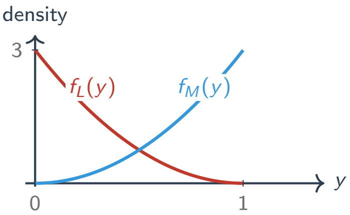
The maximum is pulled toward 1.

Minimum $L = \min X_i$

For $0 < y < 1$,

$$F_L(y) = 1 - (1-y)^3, \; f_L(y) = 3(1-y)^2.$$

The minimum is pulled toward 0.



From conditional density to joint density

Workflow

If a conditional density and a marginal density are given, then

$$f_{X,Y}(x,y) = f_{Y|X}(y \mid x) f_X(x).$$

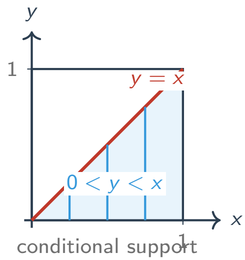
Then the marginal of Y is

$$f_Y(y) = \int f_{X,Y}(x,y)\,\mathrm{d}x,$$

with limits determined by the support.

Support first

The formula is not complete without the support. Conditional supports often depend on x.



Conditional-to-joint example

Suppose

$$f_X(x) = 2x,\ 0 < x < 1,\ f_{Y|X}(y \mid x) = \frac{1}{x},\ 0 < y < x.$$

Joint density

$$f_{X,Y}(x,y) = f_{Y|X}(y \mid x) f_X(x) = \frac{1}{x} \cdot 2x = 2,$$

for

$$0 < y < x < 1.$$

Marginal density of Y

For $0 < y < 1$, the valid values of x are $y < x < 1$:

$$f_Y(y) = \int_y^1 2\,\mathrm{d}x = 2(1-y).$$

Main lesson

The algebra is simple only after the support is written correctly.

Same triangle, two equivalent descriptions

Vertical slices

$$0 < x < 1,\ 0 < y < x.$$

Useful for integrating in the order $\mathrm{d}y\,\mathrm{d}x$.

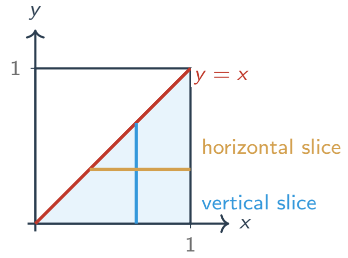
$$\int_0^1 \int_0^x f(x, y)\,\mathrm{d}y\,\mathrm{d}x.$$

Horizontal slices

$$0 < y < 1,\ y < x < 1.$$

Useful for integrating in the order $\mathrm{d}x\,\mathrm{d}y$.

$$\int_0^1 \int_y^1 f(x, y)\,\mathrm{d}x\,\mathrm{d}y.$$



Conditional binomial model

Generic hierarchical setup

Suppose

$$Y \mid P = p \sim \mathrm{Binom}(n, p).$$

Then

$$\mathbb{E}(Y \mid P = p) = np, \ \mathrm{Var}(Y \mid P = p) = np(1 - p).$$

As a random variable,

$$\mathbb{E}(Y \mid P) = nP.$$

Reading cue

If the problem first fixes p, use an ordinary binomial model. If P is random, use conditional expectation.

Law of total expectation

Formula

$$\mathbb{E}(Y) = \mathbb{E}[\mathbb{E}(Y \mid P)].$$

If

$$Y \mid P = p \sim \mathrm{Binom}(n, p),$$

then

$$\mathbb{E}(Y) = \mathbb{E}(nP) = n\mathbb{E}(P).$$

Representative example

Let $P \sim \mathrm{Unif}(0, 1)$ and $Y \mid P = p \sim \mathrm{Binom}(n, p)$. Then

$$\mathbb{E}(P) = \frac{1}{2}, \ \mathbb{E}(Y) = \frac{n}{2}.$$

Common mistake

Do not plug in one fixed value of p when P is random.

Conditional model checklist

What to write

Given	**Use**
$X_i \mid P = p \sim \mathrm{Bern}(p)$	$\mathbb{E}(X_i \mid P = p) = p$
$Y = \sum_{i=1}^{n} X_i$	$Y \mid P = p \sim \mathrm{Binom}(n, p)$
Fixed p	compute ordinary binomial probabilities
Random P	use $\mathbb{E}(Y) = \mathbb{E}[\mathbb{E}(Y \mid P)]$
Need variance	use conditional variance formulas if asked

Notation habit

Separate $\mathbb{E}(Y \mid P = p)$, a function of p, from $\mathbb{E}(Y \mid P)$, a random variable.

Central Limit Theorem setup

CLT for sample means

Let $X_1, \ldots, X_n$ be i.i.d. with

$$\mathbb{E}(X_i) = \mu, \ \mathrm{Var}(X_i) = \sigma^2 < \infty.$$

For large n,

$$\overline{X}_n \approx N\left(\mu, \frac{\sigma^2}{n}\right),$$

and

$$Z = \frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \approx N(0,1).$$

Tool distinction

CLT gives an approximation. Chebyshev gives a distribution-free guarantee. Exact normal calculations require a normal model.

CLT interval example

Suppose individual measurements have

$$\mathbb{E}(X) = 100,\ \mathrm{Var}(X) = 25.$$

For a sample of size 64, approximate

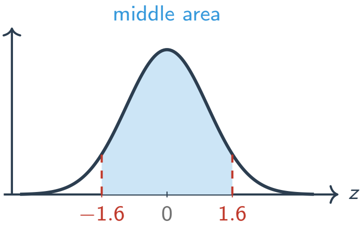
$$\Pr(|\overline{X} - 100| < 1).$$

Standard error

$$\mathrm{SD}(\overline{X}) = \frac{5}{\sqrt{64}} = 0.625.$$

$$\Pr(|\overline{X} - 100| < 1) \approx \Pr\left(|Z| < \frac{1}{0.625}\right).$$

$$= \Pr(|Z| < 1.6) = 2\Phi(1.6) - 1.$$



CLT sample size formula

Goal

Find n so that, approximately,

$$\Pr(|\overline{X}_n - \mu| < a) \geq c.$$

Using the CLT,

$$\Pr\left(|Z| < \frac{a\sqrt{n}}{\sigma}\right) \geq c.$$

If $z_{(1+c)/2}$ satisfies

$$\Pr(-z_{(1+c)/2} < Z < z_{(1+c)/2}) = c,$$

then choose

$$\frac{a\sqrt{n}}{\sigma} \geq z_{(1+c)/2}.$$

So

$$\boxed{n \geq \left(\frac{z_{(1+c)/2}\sigma}{a}\right)^2.}$$

Exact calculation, approximation, or bound?

Decision table

Information available	Use
Exact normal model	standardize and use Φ exactly
Large i.i.d. sample, finite variance	CLT approximation
Mean and variance, no distribution	Chebyshev bound
Only $X \geq 0$ and $\mathbb{E}(X)$	Markov bound
Long-run behavior of averages	Law of Large Numbers statement

Language matters

A **bound** is guaranteed under its assumptions. A **CLT calculation** is approximate. An **exact normal** calculation is exact only when the model is normal.

Two distribution-free bounds

For $X \geq 0$,

$$\Pr(X \geq t) \leq \frac{\mathbb{E}(X)}{t}.$$

For finite variance,

$$\Pr(|X - \mathbb{E}X| \geq t) \leq \frac{\mathrm{Var}(X)}{t^2}.$$

CLT standardization

$$\frac{\overline{X}_n - \mu}{\sigma/\sqrt{n}} \approx N(0, 1).$$

Convolution: sum of independent continuous variables

Formula

If $Y = X_1 + X_2$ and $X_1 \perp\!\!\!\perp X_2$, then

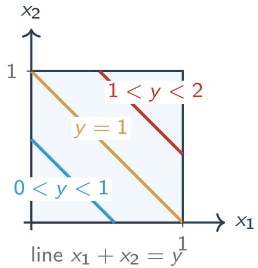
$$f_Y(y) = \int_{-\infty}^{\infty} f_1(x) f_2(y-x)\,\mathrm{d}x.$$

The integrand is nonzero only when

$$x \in \mathrm{Supp}(X_1), \;\; y - x \in \mathrm{Supp}(X_2).$$

What changes?

The integration limits usually change at breakpoints in $\mathrm{Supp}(Y)$.



Convolution example: two Uniform$(0,1)$ variables

Let $X_1, X_2 \overset{\text{iid}}{\sim} \text{Unif}(0,1)$ and $Y = X_1 + X_2$.

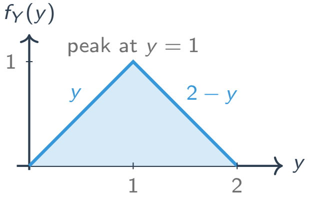
Limits

For $0 < y < 1$,

$$f_Y(y) = \int_0^y 1 \, dx = y.$$

For $1 \le y < 2$,

$$f_Y(y) = \int_{y-1}^1 1 \, dx = 2 - y.$$



Density

$$f_Y(y) = \begin{cases} y, & 0 < y < 1, \\ 2 - y, & 1 \le y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

Linearity of expectation

The most useful expectation rule

For constants a, b, c,

$$\mathbb{E}(aX + bY + c) = a\mathbb{E}(X) + b\mathbb{E}(Y) + c.$$

More generally,

$$\mathbb{E}\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i \mathbb{E}(X_i).$$

No independence needed

Linearity of expectation works even when variables are dependent.

Independence matters elsewhere

You need independence for product factorization and many variance simplifications, not for linearity.

CDFs with flat parts and jumps

General pattern

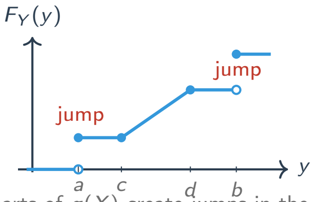
Suppose a continuous X is transformed by a rule that is constant on some intervals, for example

$$Y = \begin{cases} a, & X \le c, \\ X, & c < X < d, \\ b, & X \ge d. \end{cases}$$

Then Y may have both continuous pieces and point masses.

Reading cue

Constant parts of the transformation create jumps in the CDF of Y.



flat parts of $g(X)$ create jumps in the CDF

Quantiles from a CDF

Definition

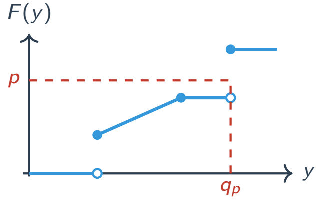
For a CDF F, a common definition of the p-quantile is

$$q_p = \inf\{y : F(y) \geq p\}.$$

This definition handles jumps correctly.

How to use it

1. Draw or read the CDF.
2. Find the first y where the CDF reaches level p.
3. If the CDF jumps over p, the quantile is the jump location.



first point where $F(y) \geq p$

High-yield mistakes to avoid

Setup mistakes

- Forgetting to state the support after a transformation.
- Treating a density value as a probability.
- Using rectangular bounds for a triangular support.
- Multiplying probabilities without independence.
- Using the CLT when the problem asks for a guaranteed bound.

Formula mistakes

- Standardizing by variance instead of standard deviation.
- Applying the one-to-one theorem to a many-to-one map.
- Dropping the absolute value in $|s'(y)|$.
- Squaring coefficients in means, or not squaring them in variances.
- Forgetting to update a Poisson mean when the interval changes.

Reliable correction

When stuck, write: random variable, support, event, formula. That usually reveals the correct next step.

Formula sheet advice

Include formulas that also remind you when to use them

Topic	What is worth writing down
Probability	conditional probability, total probability, Bayes, independence tests
Distributions	common p.m.f.s/p.d.f.s, supports, means, variances
Transformations	CDF recipe; one-to-one theorem with support check
Joint models	marginal and conditional density formulas; support sketches
Order statistics	max/min CDF and PDF formulas
Expectation	LOTUS, linearity, total expectation
Normal/CLT	standardization, sample mean variance, CLT sample-size setup
Convolution	sum formula plus "find limits from supports"
Bounds	Markov and Chebyshev statements and assumptions

Formula-sheet warning

Do not only list formulas. Add short labels such as "requires independence," "one-to-one only," or "CLT approximation."

Final checklist

Before solving

- Define the random variable and event.
- State the support or draw the region.
- Identify the tool: exact formula, transformation, conditioning, total expectation, convolution, CLT, or bound.
- Check assumptions: independence, i.i.d., normality, nonnegativity, finite variance, one-to-one transformation.
- Write the setup before simplifying arithmetic.

One-sentence strategy

Define the variable. State the support. Choose the tool. Justify the result.

How to study

Practice applying workflows to new wording. Do not memorize these examples as templates.